**Tavi 2.** **rekurentuli damokidebulebani da dinamiuri daprogrameba**

**$1. amocanis da qveamocanis cneba**

nebismieri amocanis formulirebisas aucilebelia sawyisi monacemebis gansazRvra, romlebsac amocanis parametrebi ewodeba.

magaliTad, Tu Cven vxsniT ax2+bx+c=0 kvadratuli gantolebis fesvebis povnis amocanas, maSin es amocana sami parametriT – a, b da c koeficientebiT ganisazRvreba.

Tu Cven gvsurs raime ricxvebis saSualo ariTmetikulis povnis amocanis amoxsna, maSin amocanis parametrebi ricxvTa raodenoba da maTi mniSvnelobebi iqneba.

amasTan, Cven jer amocanis amoxsnis konkretuli algoriTmi ar gvainteresebs. Cveni mizania viswavloT amocanis amoxsna misi qveamocanebis amoxsnamde dayvanis gziT. am SemTxvevaSi mosaxerxebelia algoriTmi Semavali parametrebis iseT gamomaval monacemebad gardamqmnel funqciad ganvixiloT, romlebic amocanis amonaxsns warmoadgens.

aqedan gamomdinare, zemoT aRwerili midgomis SemTxvevaSi, nebismieri amocana SesaZlebelia formalizebuli iqnas iseTi funqciis saxiT, romlis argumentebs SeiZleba warmoadgendes:

- parametrTa raodenoba;

- parametrTa mniSvnelobebi.

aqac da Semdegac parametrebad ganxiluli iqneba mTeli arauaryofiTi ricxvebi.

rogorc wesi, amocanis erT-erT arguments misi parametrTa raodenoba warmoadgens. im SemTxvevaSi, roca am parametris mniSvnelobis mixedviT sxva parametrebis konkretul mniSvnelobaTa gansazRvra SeiZleba, Cven am ukanasknelT ugulvebelvyofT. es Cveulebriv keTdeba im SemTxvevaSi, roca parametrebi cxriliT aris mocemuli. magaliTad, Tu Cven gvinda cxrilis pirveli K elementis jamis povna, maSin amocanis amosaxsnelad sakmarisia erTi K parametris codna, xolo yvela sxva parametri cxrilidan SegviZlia avirCioT.

imis Semdeg, rac amocana formalizebulia (warmodgenilia) raime argumentebis mqone funqciis saxiT, SemovitanoT ***qveamocanis*** cneba. aqac da Semdegac ***qveamocanaSi*** vigulisxmebT igive amocanas, oRond parametrTa naklebi raodenobiT, an amocanas parametrTa igive raodenobiT, magram am parametrTagan erT-erTs mainc naklebi mniSvneloba unda hqondes.

magaliTi. mocemuli 10 monetidan vipovoT maT Soris yvelaze mZime.

amocanis formalizaciisaTvis ganvsazRvroT funqcia `yvelaze mZime moneta~, romlis argumentebsac monetebis raodenoba (10) da TiToeuli maTganis masa warmoadgens. jerjerobiT Cven am funqciis konkretuli saxe ar gvainteresebs. CvenTvis umniSvnelovanes faqtors warmoadgens is, rom igi swor amonaxsns gvaZlevs.

mocemuli amocanisaTvis SegviZlia ganvixiloT 9 qveamocana, romlebsac argumentTa naklebi raodenoba gaaCnia:

`yvelaze mZime moneta~ 1 monetidan;

`yvelaze mZime moneta~ pirveli 2 monetidan;

`yvelaze mZime moneta~ pirveli 3 monetidan;

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`yvelaze mZime moneta~ pirveli 9 monetidan;

amgvarad, Cveni funqciis `yvelaze mZime moneta~ arguments warmoadgens mocemuli monetebis raodenoba, romlis mixedviT SesaZlebelia TiToeuli monetis masis gansazRvra. Sesabamisad, ganxilul qveamocanebs gaaCnia argumentTa naklebi raodenoba, vidre sawyis amocanas.

unda aRiniSnos, rom qveamocanaSi ar unda gavigoT amocanis amoxsnis zogierTi etapi, iseTebi, rogoricaa monacemTa Setanisa da gamotanis organizacia, monacemTa dalageba an dasmuli amocanis romelime nawilis amoxsna.

**$2. amocanis dayvana qveamocanebamde**

amocanaTa amoxsnis erT-erT ZiriTad meTods warmoadgens maTi iseTi qveamocanebis amoxsnamde dayvana, rom qveamocanebis amoxsnebze dayrdnobiT SesaZlebeli iyos sawyisi amocanis amonaxsnis miReba.

amasTan, sawyisi amocanis amosaxsnelad SeiZleba saWiro gaxdes erTi an ramdenime qveamocanis amoxsna.

magaliTi. amocana, romelic wina paragrafSi ganvixileT, SeiZleba sxvadasxva qveamocanaTa erTobliobebze daviyvanoT. magaliTad:

vipovoT yvelaze mZime moneta 9 monetidan, xolo Semdeg yvelaze mZime 2 monetidan (9-idan napovnsa da darCenils Soris) an

vipovoT yvelaze mZime moneta 5 monetidan, Semdeg yvelaze mZime moneta darCenili 5 monetidan, xolo amis Semdeg yvelaze mZime wina bijebze napovni 2 monetidan.

SesaZlebelia qveamocanaTa sxva erTobliobebic, magram Zneli ar aris SevniSnoT, rom yvela maTgani efuZneba erT qveamocanas: vipovoT yvelaze mZime moneta 2 monetidan.

motanil magaliTSi sawyisi amocana daiyvaneba naklebi raodenobis parametrebis mqone (monetebis naklebi raodenoba) qveamocanebze.

igive principis gamoyenebiT SesaZlebelia amovxsnaT ori ricxvis udidesi saerTo gamyofis povnis amocanac, romelic pirveli Tavis me-3 paragrafSi ganvixileT.

magaliTi. vipovoT mocemuli ori naturaluri N da M ricxvebis udidesi saerTo gamyofi.

Tu ricxvebi tolia, maSin maTi usg erT-erTi maTganis tolia, anu, usg(N, M)=N.

ganvixiloT SemTxveva, roca ricxvebi toli ar aris. cnobilia, rom

usg(N, M)=usg(N, M+N)=usg(N+M, M).

garda amisa, roca N>M, maSin usg(N, M)=usg(N\_M, M), xolo, roca M>N, maSin usg(N, M)=usg(N, M\_N).

swored ukanaskneli Tanafardobebi uzrunvelyofs amocanis amoxsnis qveamocanebamde dayvanis ZiriTad princips: erT-erTi parametris mniSvneloba gaxda ufro naklebi, Tumca maTi raodenoba ucvleli darCa.

amgvarad, usg(N, M)-is povnis amocanis amoxsna, roca N-is da M-is mniSvnelobebi gansxvavebulia, Semdeg or qveamocanaze daiyvaneba:

usg(N\_M, M), roca N>M;

usg(N, M\_N), roca M>N.

**$3. rekurentuli damokidebulebis cneba**

sawyisi amocanis amoxsnis zogierTi qveamocanis amoxsnamde dayvanis meTodi SeiZleba Caiweros damokidebulebis saxiT, romelSic sawyisi amocanis Sesabamisi funqciis mniSvneloba qveamocanebis Sesabamisi funqciebis mniSvnelobebiT gamoisaxeba. amasTan, dayvanis umniSvnelovanes pirobad iTvleba is faqti, rom damokidebulebis marjvena nawilSi Semavali nebismieri funqciis argumentTa mniSvnelobebi naklebi iyos marcxena nawilSi Semavali funqciis argumentTa mniSvnelobebze. Tu argumenti ramdenimea, maSin erT-erTi maTganis Semcirebac sakmarisia.

aqve unda aRiniSnos isic, rom damokidebuleba gansazRvruli unda iyos argumentTa yvela dasaSvebi mniSvnelobisaTvis.

magaliTi. vipovoT mocemuli A cxrilis N raodenobis elementis jami.

vTqvaT, funqcia S(N) sawyisi amocanis amonaxsns Seeesabameba. am funqcias mxolod erTi argumenti aqvs. es argumentia N, romelic A cxrilis Sesajamebeli elementebis raodenobas warmoadgens. gasagebia, rom N elementis jamis sapovnelad sakmarisia vicodeT pirveli N\_1 elementis jami da N-uri elementis mniSvneloba. amitom, sawyisi amocanis amoxsna SeiZleba Semdegi damokidebulebis saxiT Caiweros:

S(N)=S(N\_1)+aN

saWiroa aRiniSnos, rom es damokidebuleba samarTliania elementTa nebismieri N>1 raodenobisaTvis. igi SeiZleba Semdegnairad gadavweroT:

S(i)=S(i\_1)+ai,  sadac i>1

amasTan, es damokidebuleba jerjerobiT ar aris gansazRvruli N=1-saTvis. zemoT motanil damokidebulebas aucileblad unda daematos damokidebuleba S(1)=a1.

SevniSnoT, rom praqtikaSi ufro xSirad gamoiyeneba igive arsis matarebeli Semdegi damokidebulebebi:

S(i)=S(i\_1)+ai,  sadac i≥1, S(0)=0

am damokidebulebebidan pirveli maTganis mimdevrobiTi gamoyeneba i=1, 2, ..., N-saTvis gvaZlevs swored N elementis jams:

S[0]:=0

ciklis dasawyisi: i-saTvis 1-dan N-mde

S[i]:=S[i\_1]+a[i] (2.1)

ciklis dasasruli

S[i]-Si inaxeba S(i) funqciis mniSvneloba.

aqac da Semdegac mrgval frCxilebSi funqciis argumentebi Caiwereba, xolo kvadratulSi ki – masivis elementTa indeqsebi. amasTan, funqciis saxeli da masivis saxeli, romelSic am funqciis mniSvneloba inaxeba, SeiZleba erTmaneTs emTxveodes.

*SeniSvna.* S*-is indeqsi SeiZleba gamovtovoT, magram amiT damokidebulebis arsi ar icvleba. es gamowveulia imiT,* S *cxrilis Semdegi elementis gamosaTvlelad saWiroa mxolod misi wina elementis mniSvnelobis codna.*

magaliTi. gamovTvaloT jami S=1+1/x+1/x2+...+1/xN, sadac x≠0.

rogorc wina magaliTSi, aqac SeiZleba CavweroT Semdegi damokidebuleba:

S(i)=S(i\_1)+a(i), i≥1,

sadac a(i)=1/xi, S(0)=1.

ra Tqma unda, es damokidebulebebi SegviZlia gamoviyenoT programis dasawerad. amasTan, Cven unda davadginoT a(i)-is gamoTvlis meTodic, risTvisac SegviZlia visargebloT igive xerxiT – SevecadoT a(i)-is gamoTvlas a(i\_1)-is saSualebiT. a(i)-isa da a(i\_1)-is mniSvnelobebs Soris damokidebulebas eqneba saxe:

a(i)=a(i\_1)/x, i≥1, a(0)=1

aqedan gamomdinare, dasmuli amocanis amoxsnas eqneba saxe:

S[0]:=1

a[0]:=1

ciklis dasawyisi: i-saTvis 1-dan N-mde (2.2)

a[i]:=a[i\_1]/x

S[i]:=S[i\_1]+a[i]

ciklis dasasruli

*SeniSvna. unda aRiniSnos, rom am SemTxvevaSic* S*-is da a-s indeqsebi SeiZleba gamovtovoT im mosazrebiT, rom TiToeuli cxrilis Semdegi elementis gamosaTvlelad saWiroa mxolod misi wina elementis mniSvnelobis codna.*

damokidebulebebs, romlebic erTmaneTTan akavSirebs erTi da igive, magram gansxvavebuli argumentebis mqone funqciebs, ***rekurentuli damokidebulebebi*** an ***rekurentuli gantolebebi*** ewodeba.

**$4. swori rekurentuli damokidebulebebi**

***swori rekurentuli damokidebulebebi (gantolebebi)*** ewodeba iseT rekurentul damokidebulebs, romelTa marjvena nawilSi Semavali funqciebis argumentTa raodenoba an mniSvnelobebi naklebia marcxena nawilSi Semavali funqciis argumentTa raodenobaze an, Sesabamisad, mniSvnelobebze. Tu argumenti ramdenimea, maSin erT-erTi maTganis Semcirebac sakmarisia.

yuradReba unda mivaqcioT im faqts, rom damokidebulebebi gansazRvruli unda iyos argumentTa yvela dasaSvebi mniSvnelobisaTvis. amitom, unda ganisazRvros funqciaTa mniSvnelobebi parametrTa sawyisi mniSvnelobebisTvis.

zemoT ganxilul magaliTebSi damokidebulebebi erTmaneTTan akavSirebda ori gansxvavebuli parametris mqone S(i) da S(i\_1), aseve a(i) da a(i\_1) funqciebs i-s nebismieri naturaluri mniSvnelobisaTvis. amasTan, gansazRvruli iyo S(0) da a(0) sawyisi mniSvnelobebi.

unda aRiniSnos, rom am sawyisi mniSvnelobebis gareSe rekurentuli damokidebuleba

S(i)=S(i\_1)+ai,  i≥1,

araswori iqneboda, radgan igi i=1-saTvis gansazRvruli ar aris.

ra Tqma unda, SeiZleba arsebobdes ufro rTuli damokidebulebebic, romlebic erTmaneTTan orze met funqcias akavSirebs.

magaliTi. erT-erT yvelaze ufro cnobil ricxviT mimdevrobas warmoadgens fibonaCis ricxvebi, romlebic Semdegi rekurentuli damokidebulebiT ganisazRvreba:

F(0)=1,

F(1)=1,

F(i)=F(i\_1)+F(i\_2), sadac i naturaluri ricxvia da i>1

am SemTxvevaSi, F(N)-is mniSvnelobis gamosaTvlelad (romelic F[N]-Si inaxeba) SegviZlia Semdegi algori|TmiT visargebloT:

F[0]:=1

F[1]:=1

ciklis dasawyisi: i-saTvis 2-dan N-mde (2.3)

F[i]:=F[i\_1]+F[i\_2]

ciklis dasasruli

*SeniSvna. motanil fragmentSi indeqsebis ugulvebelyofa ukve aRar SeiZleba. Tumca, principSi SesaZlebelia F(N)-is mniSvnelobis gamoTvla cxrilis gamoyenebis gareSec, magram es ukve algoriTmis realizaciis meTodis SerCevis sakiTxs warmoadgens.*

*qvemoT motanilia aseTi realizaciis magaliTi:*

*a:=1*

*b:=1*

*ciklis dasawyisi: i-saTvis 2-dan N-mde*

*c:=a+b (2.4)*

*a:=b*

*b:=c*

*ciklis dasasruli*

*aq gamoiyeneba is faqti, rom cxrilis mimdinare elementis gamosaTvlelad sakmarisia mxolod misi wina ori elementis mniSvnelobis codna.*

**$5. cxrilebis organizaciis meTodi**

Cvens mier ganxiluli magaliTebidan Cans, rom amocanis amoxsnis dros umniSvnelovanes moments warmoadgens am amocanis qveamocanebamde dayvanis meTodi. Tumca, aranakleb mniSvnelovania qveamocanebis amoxsnebidan sawyisi amocanis amoxsnis agebis meTodic. erT-erT aseT efeqtur saSualebas qveamocanebis amoxsnaTa dasamaxsovreblad cxrilebis gamoyeneba wamoadgens. amocanebis amoxsnis aseT meTods ***dinamiuri daprogramebis*** meTodi ewodeba.

rogorc zemoT iyo aRniSnuli, qveamocana SeiZleba formalizebuli iqnas funqciis saxiT, romelsac erTi an ramdenime argumenti gaaCnia. Tu Cven aviRebT cxrils, romlis elementebis raodenoba funqciis argumentebis yvela SesaZlo gansxvavebul nakrebTa raodenobis tolia, maSin SegviZlia argumentTa TiToeul nakrebs cxrilis elementi SevusabamoT. gamovTvliT ra cxrilis elementebs (qveamocanebis amonaxsnebs), SevZlebT vipovoT sawyisi amocanis amonaxsnic.

**5.1. erTganzomilebiani cxrilebis organizacia**

cxrilebis organizaciis erT-erT meTods warmoadgens iseTi meTodi, roca cxrilis zoma amocanis Sesabamisi funqciis argumentebis raodenobiT ganisazRvreba.

magaliTi. vipovoT mocemuli 10 elementiani A cxrilis elementTa namravli.

vTqvaT, funqcia P(10) sawyisi amocanis amonaxsns Seesabameba. mocemul SemTxvevaSi funqcias mxolod erTi argumenti (elementTa raodenoba) aqvs. 10 elementis namravlis sapovnelad sakmarisia vicodeT 9 elementis namravli da me-10 elementis mniSvneloba. amitom, sawyisi amocanis amoxsna SegviZlia CavweroT Semdegi damokidebulebis saxiT: P(10)=P(9)⋅a10. aqedan gamomdinare, es damokidebuleba SeiZleba gansazRvruli iqnas nebismieri i-saTvis, sadac 2≤i≤10:

P(i)=P(i\_1)⋅ai da P(1)=a1

praqtikaSi, ganxiluli amocanisaTvis ufro xSirad gamoiyeneba rekurentuli damokidebuleba, romelic arsiT igivea, magram sxva sawyisi mniSvneloba aqvs:

P(i)=P(i\_1)⋅ai, sadac i≥1, P(0)=1

algoriTms, romelic realizacias ukeTebs mocemul rekurentul damokidebulebas, aqvs saxe:

P[0]:=1

ciklis dasawyisi: i-saTvis 1-dan N-mde (2.5)

P[i]:=P[i\_1]\*a[i]

ciklis dasasruli

radganac Cvens funqcias erTi argumenti (TanamamravlTa raodenoba) aqvs, amitom amocanis amosaxsnelad sakmarisia erTganzomilebiani cxriliT visargebloT. amasTan, cxrilis elementTa raodenoba argumentis gansxvavebul mniSvnelobaTa raodenobiT ganisazRvreba. ganxilul magaliTSi masivis ganzomileba 10-is tolia. Tu Cven amovxsnidiT 20 elementis namravlis povnis amocanas, maSin rekurentuli damokidebulebis sarealizaciod 20 elementiani, erTganzomilebiani cxrili dagvWirdeboda.

amgvarad, rekurentuli damokidebulebis sarealizaciod saWiro cxrilis zoma qveamocanebis Sesabamis funqciaTa argumentebis raodenobiT ganisazRvreba, xolo TiToeuli ganzomilebis mixedviT (striqonebSi da svetebSi) elementTa raodenoba ki - Sesabamisi argumentis SesaZlo mniSvnelobaTa raodenobiT.

**5.2. organzomilebiani cxrilebis organizacia**

kidev erTxel mivaqcioT yuradReba imas, rom Cven jer ar gvainteresebs algoriTmis realizacia, romlis drosac xdeba iseTi maxasiaTeblis minimizacia, rogoric gamoyenebuli operatiuli mexsierebaa. vigulisxmoT, rom Sesabamisi cxrilis Sesanaxad kompiuters sakmarisi mexsiereba aqvs.

magaliTi. 5X6 zomis mocemuli marTkuTxa A cxrilisaTvis avagoT igive zomis iseTi marTkuTxa **B** cxrili, romlis elementebic Sedegnairad SeirCeva: B[i, j] elementi maqsimaluris tolia A cxrilis im elementebs Soris, romlebic (i, j) poziciis marcxniv da zemoTaa ganlagebuli ((i, j) poziciis CaTvliT). amasTan, iTvleba, rom pozicia (1, 1) marTkuTxa cxrilis zeda marcxena poziciaa. cxrilis CvenTvis saintereso nawili gamoyofilia, roca i=3 da j=4.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a11 | a12 | a13 | a14 | a15 | a16 |
| a21 | a22 | a23 | a24 | a25 | a26 |
| a31 | a32 | a33 | a34 | a35 | a36 |
| a41 | a42 | a43 | a44 | a45 | a46 |
| a51 | a52 | a53 | a54 | a55 | a56 |

T(i, j)-iT avRniSnoT funqcia, romelic B(i, j) elements gamoiTvlis.

Tavidan ganvsazRvroT B cxrilis im elementebis mniSvnelobebi, romlebic pirvel striqonSi da pirvel svetSia ganlagebuli. miviRebT:

T(1, 1)=A[1, 1];

T(1, j)=max{T(1, j\_1), A[1, j]}, roca j≥2;

T(i, 1)=max{T(i\_1, 1), A[i, 1]}, roca i≥2.

es damokidebulebebi gamomdinareobs iqedan, rom am SemTxvevaSi A matricis CvenTvis saintereso nawili SemosazRvrulia am matricis mxolod pirveli striqonis an pirveli svetis elementebiT.

roca 2≤i≤5 da 2≤j≤6, maSin am funqciisaTvis SegviZlia CavweroT Semdegi rekurentuli damokidebuleba:

T(i, j)=max{T(i\_1, j), T(i, j\_1), A[i, 1]}

marTlac, sidide T(i\_1, j) Seesabameba maqsimalur elements A cxrilis im nawilSi, romelic i\_1 da j indeqsebis mniSvnelobebiT ganisazRvreba, xolo T(i, j\_1) ki - maqsimalur elements A cxrilis im nawilSi, romelic i da j\_1 indeqsebis mniSvnelobebiT ganisazRvreba. amitom, es sidideebi iTvaliswinebs A matricis yvela elementis mniSvnelobas mis im nawilSi, romelic ganisazRvreba i da j indeqsebis mniSvnelobebiT erTi A[i, j] elementis gamoklebiT.

radganac T funqcias ori argumenti aqvs, amitom am rekurentuli damokidebulebebis sarealizaciod sakmarisia organzomilebiani (marTkuTxa) cxrili. mniSvnelovania aRiniSnos, rom Cven SegviZlia T(i, j) da B[i, j] sidideebi gavaigiveoT. maSin, Sesabamisi algoriTmis fragmenti SeiZleba Semdegi saxiT Caiweros:

B[1, 1]:=A[1, 1]

ciklis dasawyisi: j-saTvis 2-dan 6-mde

B[1, j]:= max(B[1, j\_1], A[1, j])

ciklis dasasruli

ciklis dasawyisi: i-saTvis 2-dan 5-mde

B[i, 1]:= max(B[i\_1, 1], A[i, 1])

ciklis dasasruli (2.6)

ciklis dasawyisi: i-saTvis 2-dan 5-mde

ciklis dasawyisi: j-saTvis 2-dan 6-mde

B[i, j]:= max(B[i, j\_1], B[i\_1, j])

B[i, j]:= max(B[i, j], A[i, j])

ciklis dasasruli

ciklis dasasruli

**$6. cxrilis elementebis gamoTvlis meTodi**

imis Semdeg, rac amocana qveamocanebzea dayvanili da gansazRvrulia am dayvanis Sesabamisi rekurentuli damokidebulebebi, saWiroa ganisazRvros cxrilis elementebis gamoTvlis metnaklebad racionaluri meTodi.

**6.1. erTganzomilebiani cxrilis elementebis gamoTvla**

# Cveulebriv, erTganzomilebiani cxrilisaTvis aseT meTods warmoadgens misi elementebis mimdevrobiTi gamoTvla, dawyebuli pirvelidan.

magaliTi. gansazRvreT, ramdeni gansxvavebuli gziT SeiZleba kibis me-10 safexurze moxvedra, Tu erT bijze SesaZlebelia momdevno an erTis gamotovebiT mdebare safexurze asvla.

vTqvaT, K(10) kibis me-10 safexurze mosaxvedrad saWiro gzaTa raodenobis povnis amocanaa. ganvsazRvroT Cveni amocanis i-uri qveamocana, rogorc i-ur safexurze mosaxvedrad saWiro gzaTa raodenobis povnis amocana.

amocanis pirobidan gamomdinare, me-10 safexurze moxvedra SesaZlebelia uSualod me-8 da me-9 safexurebidan. amitom, Tu Cven viciT me-8 da me-9 safexurebze mosaxvedrad saWiro gzaTa K(8) da K(9) raodenobebi, maSin me-10 safexurze mosaxvedrad saWiro gzaTa raodenoba Semdegnairad ganisazRvreba: K(10)=K(8)+K(9).

aseTi damokidebuleba miiReba imitom, rom me-8 safexurze mosaxvedrad saWiro nebismieri gza me-9 safexurze gadabijebis Sedegad me-10 safexurze mosaxvedrad saWiro gzad gardaiqmneba. aseve, me-9 safexurze mosaxvedrad saWiro nebismieri gza me-9dan me-10 safexurze asvliT me-10 safexurze mosaxvedrad saWiro gzad gardaiqmneba. garda amisa, yvela es gza gansxvavebulia.

analogiuri damokidebuleba samarTliania nebismieri i-uri safexurisaTvis, dawyebuli mesamedan:

K(i)=K(i\_2)+K(i\_1)

dasadgeni darCa K(1)-is da K(2)-is mniSvnelobebi, romlebic tolia: K(1)=1, K(2)=2.

aqedan gamomdinare, amocanis amosaxsnelad sakmarisia 10 elementiani erTganzomilebiani cxrili, romlisTvisac saWiroa misi elementebis mimdevrobiT gamoTvla zemoT motanili rekurentuli damokidebulebebis Sesabamisad:

K[1]:=1

K[2]:=2

ciklis dasawyisi: i-saTvis 3-dan 10-mde (2.7)

K[i]:=K[i\_1]+K[i\_2]

ciklis dasasruli

**6.2. organzomilebiani cxrilis elementebis gamoTvla**

magaliTi. NXM zomis cxrilSi, romlis elementebi mxolod 0-ebi da 1-ebia, vipovoT maqsimaluri zomis iseTi kvadratuli bloki, romelic mxolod erTianebisagan Sedgeba. blokSi igulisxmeba cxrilis mezobeli (mimdevrobiT ganlagebuli) striqonebisa da svetebis elementTa simravle. cxrilis CvenTvis saintereso nawili gamoyofilia.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |

nebismieri kvadratuli blokis mdebareoba SeiZleba gansazRvruli iqnas misi zomiT da erT-erTi kuTxis poziciiT.

SemovitanoT T(i, j) funqcia. davuSvaT, rom misi mniSvneloba Seesabameba mxolod erTianebis Semcveli maqsimaluri kvadratuli blokis zomas, romlis marjvena qveda kuTxec (i, j) poziciaSi mdebareobs. T(i, j) funqcia B[i, j] cxrilis elements gamoiTvlis. zemoT motanili cxrilisaTvis T(i, j) funqciis mniSvnelobebs eqneba saxe:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i\j | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 2 | 2 | 0 | 1 |
| 3 | 1 | 1 | 2 | 3 | 1 | 1 |
| 4 | 1 | 2 | 0 | 1 | 2 | 2 |
| 5 | 1 | 0 | 1 | 1 | 0 | 1 |

amgvarad, Cveni amocana dayvanili iqna T(i, j) funqciis maqsimaluri mniSvnelobis gamoTvlaze i da j parametrebis yvela SesaZlo mniSvnelobisaTvis. am funqcias SeiZleba SevusabamoT cxrili zomiT N⋅M.

Tavidan ganvsazRvroT B cxrilis im elementTa mniSvnelobebi, romlebic pirvel striqonSi da pirvel svetSia ganlagebuli. miviRebT:

B[1, 1]=A[1, 1];

B[1, j]=A[1, j], roca j≥2;

B[i, 1]=A[i, 1], roca i≥2.

es damokidebulebebi gamomdinareobs iqedan, rom am SemTxvevebSi A matricis ganxiluli nawili mis mxolod erT elements Seicavs.

roca 2≤i≤N da 2≤j≤M, maSin am funqciisaTvis SegviZlia CavweroT Semdegi rekurentuli damokidebulebebi:

B[i, j]=0, Tu A[i, j]=0

da

B[i, j]=min{B[i\_1, j], B[i, j\_1], B[i\_1, j\_1]}+1, Tu A[i, j]=1

pirveli damokidebuleba gviCvenebs, rom im maqsimaluri erTeulovani blokis zoma, romlis marjvena qveda kuTxe (i, j) poziciaSi mdebareobs, nulis tolia, roca A[i, j]=0.

davrwmundeT meore damokidebulebis sisworeSi. marTlac, B[i\_1, j] sidide A cxrilis im erTeulovani blokis maqsimalur zomas Seesabameba, romlis marjvena qveda kuTxe (i\_1, j) poziciaSi mdebareobs. maSin im erTeulovani blokis zoma, romlis marjvena qveda kuTxe (i, j) poziciaSia, ar aRemateba B[i\_1, j]+1 sidides, radganac bloks poziciaSi (i\_1, j) mxolod erTi striqoni SeiZleboda damateboda.

B[i, j\_1] sidide A cxrilis im erTeulovani blokis maqsimalur zomas Seesabameba, romlis marjvena qveda kuTxe (i, j\_1) poziciaSi mdebareobs. maSin im erTeulovani blokis zoma, romlis marjvena qveda kuTxe (i, j) poziciaSia, ar aRemateba B[i, j\_1]+1 sidides, radganac bloks poziciaSi (i, j\_1) mxolod erTi sveti SeiZleboda damateboda.

B[i\_1, j\_1] sidide A cxrilis im erTeulovani blokis maqsimalur zomas Seesabameba, romlis marjvena qveda kuTxe (i\_1, j\_1) poziciaSi mdebareobs. maSin im erTeulovani blokis zoma, romlis marjvena qveda kuTxe (i, j) poziciaSia, ar aRemateba B[i\_1, j\_1]+1 sidides, radganac bloks poziciaSi (i\_1, j\_1) mxolod erTi striqoni da erTi sveti SeiZleboda damateboda.

amgvarad, im erTeulovani blokis zoma, romlis marjvena qveda kuTxe (i, j) poziciaSi mdebareobs, tolia: min{B[i\_1, j], B[i, j\_1], B[i\_1, j\_1]}+1. Sesabamis algoriTms eqneba saxe:

B[1, 1]:=A[1, 1]

ciklis dasawyisi: j-saTvis 2-dan 6-mde

B[1, j]:= A[1, j]

ciklis dasasruli

ciklis dasawyisi: i-saTvis 2-dan 5-mde

B[i, 1]:= A[i, 1]

ciklis dasasruli (2.8)

ciklis dasawyisi: i-saTvis 2-dan 5-mde

ciklis dasawyisi: j-saTvis 2-dan 6-mde

Tu A[i, j]=1

maSin

B[i, j]:= min(B[i, j\_1], B[i\_1, j])

B[i, j]:= min(B[i, j], B[i\_1, j\_1])+1

dasasruli

ciklis dasasruli

ciklis dasasruli

**6.3. organzomilebiani cxrilis elementebis gamoTvla damatebiTi SezRudvebiT**

magaliTi. sawyobSi inaxeba 5 nivTi. TiToeuli maTganisaTvis cnobilia misi Rirebuleba (larebSi) da masa (kilogramebSi). Rirebulebebi da masebi naturaluri ricxvebiT gamoisaxeba. Cveni mizania ganvsazRvroT im nivTebis maqsimaluri jamuri Rirebuleba, romelTa sawyobidan waRebac SesaZlebelia im pirobiT, rom maTi jamuri wona ar aRematebodes 16 kilograms.

vTqvaT, C cxrilis Ci elementi i-uri nivTis Rirebulebas Seesabameba, xolo M cxrilis Mi-uri elementi ki – i-uri nivTis masas. CavTvaloT, rom nivTebi gadanomrilia cxrilebSi maTi ganlagebis rigis mixedviT.

SemovitanoT T funqcia. davuSvaT, rom misi mniSvneloba Cveni amocanis amonaxsns Seesabameba. am funqciis argumentebs warmoadgens nivTebis raodenoba (am argumentiT SeiZleba ganisazRvros Sesabamis nivTTa Rirebulebebi da masebi) da is maqsimaluri masa, romlis waRebac SesaZlebelia.

ganvsazRvroT T(i, j) qveamocanebi Cveni T(5, 16) amocanisaTvis. aq i aris im sawyisi nivTebis raodenoba, romlebidanac unda moxdes wasaRebi nivTebis amorCeva, xolo j – am wasaRebi nivTebis maqsimaluri SesaZlebeli jamuri masa. unda aRiniSnos, rom amgvarad Semotanili pirveli i parametri qveamocanisaTvis gansazRvravs rogorc nivTebis raodenobas, aseve maTi Rirebulebebisa da masebis mniSvnelobebsac C da M cxrilebidan.

Tavidan davadginoT T funqciis sawyisi mniSvnelobebi. erT-erTi argumentis nulovani mniSvnelobisaTvis funqciis mniSvnelobac nulis tolia:

T(0, 0)=0;

T(0, j)=0, roca j≥1;

T(i, 0)=0, roca i≥1.

davadginoT T(i, j) funqciis SesaZlo mniSvnelobebi argumentTa aranulovani mniSvnelobebisaTvis.

T(i, j) funqciis Sesabamisi qveamocanis amoxsna SeiZleba dayvanili iqnas or SesaZleblobamde: xvdeba wasaRebi nivTebis siaSi (saukeTeso amoxsnaSi) i nomris mqone nivTi Tu ara.

Tu ar xvdeba, maSin amocanis amoxsna i raodenobis nivTisaTvis daiyvaneba qveamocanis amoxsnaze i\_1 raodenobis nivTisaTvis, anu:

T(i, j)=T(i\_1, j)

Tu i nomris mqone nivTi wasaRebi nivTebis siaSi xvdeba, maSin pirveli i\_1 nivTisaTvis maqsimaluri SesaZlo jamuri masa mcirdeba M[i]-iT da, amavdroulad, darCenili nivTebisaTvis T(i\_1, j\_M[i]) amonaxsni izrdeba C[i]-iT, anu:

T(i, j)= T(i\_1, j\_M[i])+ C[i]

amasTan, aucileblad unda gaviTvaliswinoT is faqti, rom meore situacia SesaZlebelia mxolod im SemTxvevaSi, roca i-uri nivTis masa j-s mniSvnelobas ar aRemateba.

amis Semdeg, saukeTeso amonaxsnis misaRebad Cven unda avarCioT am ori SesaZleblobidan ukeTesi. aqedan gamomdinare, rekurentul damokidebulebas (roca i≥1 da j≥1) eqneba saxe:

T(i, j)=T(i\_1, j), roca j <M[i],

T(i, j)=max{ T(i\_1, j), T(i\_1, j\_M[i])+ C[i]}, roca j ≥M[i]

vTqvaT, 5 nivTisaTvis mocemulia maTi Rirebulebebisa da masebis Semdegi mniSvnelobebi:

C[1]=5, M[1]=4;

C[2]=7, M[2]=5;

C[3]=4, M[3]=3;

C[4]=9, M[4]=7;

C[5]=8, M[5]=6.

T funqciis mniSvnelobaTa cxrili, romelsac aseve T-Ti avRniSnavT, Semdegnairad gamoiyureba:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| i\j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2 | 0 | 0 | 0 | 0 | 5 | 7 | 7 | 7 | 7 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| 3 | 0 | 0 | 0 | 4 | 5 | 7 | 7 | 9 | 11 | 12 | 12 | 12 | 16 | 16 | 16 | 16 | 16 |
| 4 | 0 | 0 | 0 | 4 | 5 | 7 | 7 | 9 | 11 | 12 | 13 | 14 | 16 | 16 | 18 | 20 | 21 |
| 5 | 0 | 0 | 0 | 4 | 5 | 7 | 7 | 9 | 11 | 12 | 13 | 15 | 16 | 17 | 19 | 20 | 21 |

Sesabamisad, amocanis amonaxsni T(5, 16)=21, anu SesaZlebelia 21 laris Rirebulebis nivTebis waReba.

Sesabamis algoriTms eqneba saxe:

T[0, 0]:=0

ciklis dasawyisi: j-saTvis 1-dan 16-mde

T[0, j]:=0

ciklis dasasruli

ciklis dasawyisi: i-saTvis 1-dan 5-mde

T[i, 0]:=0

ciklis dasasruli (2.9)

ciklis dasawyisi: i-saTvis 1-dan 5-mde

ciklis dasawyisi: j-saTvis 1-dan 16-mde

Tu j>=M[i]

maSin

T[i, j]=max( T[i\_1, j], T[i\_1, j\_M[i]]+ C[i])

Tu arada

T[i, j]=T[i\_1, j]

dasasruli

ciklis dasasruli

ciklis dasasruli